A COMPUTATIONAL FRAMEWORK FOR MULTI-MATERIAL FLUID-STRUCTURE INTERACTION WITH DYNAMIC FRACTURE

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**MOTIVATION**

- FSI problems with dynamic fracture

**pipeline explosion**

**underwater explosion & implosion**

- shock waves at/near material interfaces
- large structural deformation
- fluid-induced fracture and/or fragmentation
- flow seepage through cracks
- multi-phase flow with large density/pressure jump
- multiple time scales

**computational challenges**

**shock lithotripsy**
Multi-material compressible fluids in Eulerian formulation

- Euler equations
  \[
  \frac{\partial W}{\partial t} + \nabla \cdot \mathbf{F}(W) = 0
  \]
  where \( W = \begin{bmatrix} \rho \\ \rho \nu \\ E \end{bmatrix} \),

  and \( \mathbf{F} = [\rho \nu, \nu \otimes (\rho \nu) + p I, (E + p) \nu] \)

- Immiscible fluids / free surface
  \[
  \frac{\partial \varphi}{\partial t} + \nu \cdot \nabla \varphi = 0 \quad \text{(level-set equation for interface tracking)}
  \]
  \[
  \nu_L \cdot \mathbf{n} = \nu_R \cdot \mathbf{n}, \quad p_L = p_R
  \]

- Equations of state (EOS)
  \[
  p = (\gamma_s - 1) \rho e - \gamma_s \rho s
  \]
  (stiffened gas for liquids)

  \[
  p = A(1 - \frac{\omega \rho}{R_1 \rho_0}) e^{-\frac{R_1 \rho_0}{\rho}} + B(1 - \frac{\omega \rho}{R_2 \rho_0}) e^{-\frac{R_2 \rho_0}{\rho}} + \omega \rho e
  \]
  (JWL for explosion products)
Elastic-plastic shells in Lagrangian formulation

- equation of motion
\[ \rho_s \ddot{u}_s - \nabla \cdot \sigma_s(u_s, \dot{u}_s) = f^{ext} \]

- 2nd-order Green’s strain tensor
\[ \varepsilon_s = \frac{1}{2} (\nabla u_s + \nabla u_s^T + \nabla u \otimes \nabla u^T) \]

- J_2 plasticity, piecewise linear hardening
- EPS-based failure criterion

Fluid-structure interaction

- Impermeable interface ➔ fluid-structure transmission conditions (for inviscid fluid flows)
\[ \nu \cdot n_s = \dot{u}_s \cdot n_s \quad \text{“no interpenetration”} \]
\[ -pn_s = \sigma_s \cdot n_s \quad \text{“equilibrium”} \]
The standard finite volume semi-discretization

- Euler equations

\[ \frac{\partial W}{\partial t} + \nabla \cdot F(W) = 0 \]

- integrate over a control volume \((C_i)\)

\[ \int_{C_i} \frac{\partial W_h}{\partial t} d\Omega + \sum_{j \inenv(i) \partial C_{ij}} \int F(W_h) \cdot \vec{n}_{ij} d\Gamma = 0 \]

- evaluate one numerical flux for each “facet” \((\partial C_{ij})\)

\[ \int_{\partial C_{ij}} F(W_h) \cdot \vec{n}_{ij} d\Gamma = Roe(W_h^i, W_h^j, \vec{n}_{ij}, EOS) \]

- special treatment is required near material (i.e. fluid-fluid or fluid-structure) interfaces
FIVER AT FLUID-STRUCTURE INTERFACE

1D fluid-structure Riemann problem

\[ \frac{\partial W}{\partial \tau} + \frac{\partial F(W)}{\partial \xi} = 0 \]

\[ W(\xi,0) = W_{ij} \]

\[ V(0,\tau) = \hat{u}_s \cdot n_s \]

\[ \sum_{F/S} \text{(embedded surface)} \]

\[ \frac{\partial W}{\partial \tau} + \frac{\partial F(W)}{\partial \xi} = 0 \]

\[ W(\xi,0) = W_{ji} \]

\[ V(0,\tau) = \hat{u}_s \cdot n_s \]

\[ W^*_i \]

\[ W^*_j \]

* Wang, Rallu, Gerbeau, Farhat 2011
Fluid-fluid Riemann problem

\[ \frac{\partial W}{\partial \tau} + \frac{\partial F(W)}{\partial \xi} = 0 \]

\[ W(\xi,0) = \begin{cases} W_{ij} & \text{if } \xi < 0 \\ W_{ji} & \text{if } \xi \geq 0 \end{cases} \]

- 2nd order accuracy at interfaces can be obtained by linear extrapolation (Zeng, Main, Farhat, in press)
VALIDATION EXAMPLES

- Underwater implosion

- Flapping wings

(Farhat, Wang, et. al. IJSS, 2013)

(Farhat, Lakshminarayan, JCP, 2014)
FSI WITH DYNAMIC FRACTURE

- **FEM-based computational methods for fracture**
  - element deletion
    - delete element if failure criterion (e.g. EPS) is met
    - ☀ robust and easy to implement
    - ☹ no defined crack path
    - ☹ loss of mass, momentum, energy
  - extended finite element method (XFEM)
    - enrich the FE basis locally with step functions
      \[
      u^h(X, t) = \sum_i N_i(X)\{u_i(t) + H(f(X))q_i\}
      \]
    - enrichment \((q_i)\) is injected when a criterion for crack nucleation or growth is met
    - ☀ explicitly tracks crack propagation
    - ☹ difficult to track multiple interacting cracks
  - cohesive elements, remeshing, etc.
  - “generic” FSI methods are desirable
A generic representation of fractured interfaces

- **phantom elements**
  - for element deletion: deleted elements
  - for XFEM: duplicate elements near crack
  - can be easily extended to many other FE-based fracture methods

- **augmented fluid-structure interface \( \Sigma_{F/S}^* \)**
  - real elements + phantom elements

- **level-set function \( \phi \)** defined in \( \Sigma_{F/S}^* \)
  - gives signed distance to the closest crack
  - implicit representation of crack: \( \{ \mathbf{X} \in \Sigma_{F/S}^* \mid \phi(\mathbf{X}) = 0 \} \)
  - implicit representation of fractured interface: \( \{ \mathbf{X} \in \Sigma_{F/S}^* \mid \phi(\mathbf{X}) > 0 \} \)
  - in practice, the precise value of \( \phi \) is only needed near cracks

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* originally proposed by Song and Belytschko (2006) to simplify the implementation of XFEM in legacy FE codes
- an intersection based algorithm* is used to track the embedded fluid-structure interface with respect to the non-body-fitted CFD grid
- an intersection point $x^*$ in $\Sigma^*_{F/S}$ is accepted if and only if $\phi(x^*) > 0$

* Wang, Gretarsson, Main, Farhat, IJNMF, 2012
PIPE FRACTURE DUE TO DETONATION

Fracture of aluminum pipe due to internal detonation

* Performed by J. Shepherd et al. at California Institute of Technology.*
EXPERIMENTAL RESULT

- Crack propagation patterns

  *Direction of detonation wave*

  **Shot 120**: forward and backward curving, in the same direction

  **Shot 143**: forward and backward curving, in opposite directions

  **Shot 147**: forward and backward bifurcation
EXPERIMENTAL RESULT

Blast pressure: the notch length series

- Initial notch length (mm)
- Peak blast pressure (kPa)

Graph showing the relationship between initial notch length and peak blast pressure.
Simulation setup

- detonation modeled using the Chapman-Jouguet theory

\[
\rho_0 (U_{CJ} - u_0) = \rho_{CJ} (U_{CJ} - u_{CJ})
\]
\[
p_0 + p_0 (U_{CJ} - u_0)^2 = p_{CJ} + \rho_{CJ} (U_{CJ} - u_{CJ})^2
\]
\[
e_0 + \frac{p_0}{\rho_0} + \frac{1}{2} (U_{CJ} - u_0)^2 = e_{CJ} + \frac{p_{CJ}}{\rho_{CJ}} + \frac{1}{2} (U_{CJ} - u_{CJ})^2
\]

- three fluid materials:
  explosive gas (C_2H_2 + O_2), detonation product, and water
- CFD grid: 1.4M nodes, 8.5M elements(on 100~200 proc. cores)
- initial notch: 25.4mm / 38.1mm / 50.8mm / 63.5mm / 76.2mm
Modeling the initial notch

- XFEM: notch is modeled as an initial crack
- Element deletion: notch is modeled by shells with reduced thickness
3D Visualization of simulation result

Pressure Contours

Domain of Detonation Product (Material Id: 2)

Time: 0.000000

Output Along Pipe Axis

simulation with XFEM
SIMULATION RESULT

- Structural deformation, stress, and strain

Structural Deformation

Effective Plastic Strain

Effective Stress (Pa)

Time: 0.000000

Simulation with XFEM
SIMULATION RESULT

- 2D cut-view of fluid pressure field

![2D Cut-view at y=0](image)

Simulation with XFEM
Fracture response

- XFEM: curving in the same direction or opposite directions
- element deletion: branching
- all these propagation patterns are observed in experiment

![Fracture simulation images]

XFEM, L= 25.4 mm

XFEM, L= 38.1 mm

ED, L= 38.1 mm
VALIDATION

- Blast pressure

  summary
  • simulations captured the crack propagation modes
  • linear trend in peak pressure vs. notch length is captured
  • discrepancy in peak blast pressure: 5~20%