PREDICTIVE SIMULATION OF UNDERWATER IMPLOSION: Coupling Multi-Material Compressible Fluids with Cracking Structures

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MOTIVATION

- Underwater explosion and implosion

**Multi-Material Fluid-Structure Interaction with Dynamic Fracture**
PROBLEM CHARACTERISTICS

- Implosive collapse of underwater structures

- Large, plastic structural deformation
- Fluid-induced fracture
- Flow seepage → multi-phase flow
- High compressions and shocks in water/air

* courtesy of McGarity, O., NSWC Carderock
Euler fluid flows, Eulerian formulation
\[
\frac{\partial W}{\partial t} + \mathbf{r} : \mathbf{F}(W) = 0
\]
- water: stiffened gas or Tait EOS

Impermeable fluid-structure interface
\[
v_f \cdot n = v_s \cdot n \quad \text{“no-interpenetration”}
\]
\[-p \cdot n = \sigma_s \cdot n \quad \text{“equilibrium”}
\]

Free surface / immiscible fluids
\[
v_L \cdot n_L = v_R \cdot n_R , \quad p_L = p_R
\]
\[
\frac{\partial \phi}{\partial t} + \mathbf{v} : \mathbf{r} \phi = 0 \quad \text{(level-set)}
\]

Lagrangian equations of motion
\[
\rho_s \frac{\partial^2 u_j}{\partial t^2} = \frac{\partial}{\partial x_i} \left( \sigma_{ij} + \sigma_{im} \frac{\partial u_j}{\partial x_m} \right) + b_j , j = 1,2,3
\]
- J2 plasticity, piecewise linear hardening
- strain based failure criteria

MATHEMATICAL MODEL

determination of crack growth
(J-H Song, 2008)
Arbitrary Lagrangian-Eulerian (ALE)

- relatively simple treatment of material interfaces
- accuracy and numerical stability issues well understood

- lack of robustness with respect to large deformations
- cannot handle topological changes
- mesh motion computation can be expensive
**Embedded / immersed boundary method**

- operates on fixed, non body-fitted CFD grids

\[
\begin{align*}
\n \bf{v}_f \cdot \bf{n} &= \bf{v}_s \cdot \bf{n} \\
\text{- no interpenetration} \\
\text{- equilibrium} \\
\n-\rho \ \bf{n} &= \bf{\sigma}_s \cdot \bf{n}
\end{align*}
\]

- dramatically simplifies mesh generation
- capable of large structural deformations and fracture
- interface needs to be tracked with respect to CFD grid
- enforcing the transmission conditions becomes tricky

- various names: immersed, Cartesion, fictitious domain, etc.
The standard finite volume spatial discretization

- Euler equations

\[ \frac{\partial W}{\partial t} + \nabla \cdot F(W) = 0 \]

- integrate over a control volume \((C_i)\)

\[ \int \frac{\partial W_h}{\partial t} d\Omega + \sum_{j \in \text{nei}(i)} \int F(W_h) \cdot \vec{n}_{ij} d\Gamma = 0 \]

- evaluate one numerical flux for each “facet” \((\partial C_{ij})\)

\[ \int_{\partial C_{ij}} F(W_h) \cdot \vec{n}_{ij} d\Gamma = Roe(W_h^i, W_h^j, \vec{n}_{ij}, EOS) \]

- special treatment is required near fluid-structure and fluid-fluid interfaces
- \(<\text{fluid 1, structure}> : \text{fluid-structure Riemann problem}\)
One-dimensional, fluid-structure Riemann problem

\[ \frac{\partial w}{\partial \tau} + \frac{\partial F}{\partial \xi} (w) = 0 \]

\[ w(\xi,0) = W_L^n, \quad \text{if} \quad \xi \leq 0 \]

\[ u(x(\tau), \tau) = u_s(M_{ij}) \cdot n_\Gamma \]

\( ^\dagger \) could also be a shock
- <fluid 1, structure> : fluid-structure Riemann problem $\rightarrow W_i^*$
- <fluid 2, structure> : fluid-structure Riemann problem $\rightarrow W_j^*$

$\Phi_{ij}^{(1)} = \text{Roe} (W_i, W_i^*, \text{EOS}^{(1)})$ \hspace{1cm} (fluid 1, structure)

$\Phi_{ij}^{(2)} = \text{Roe} (W_j, W_j^*, \text{EOS}^{(2)})$ \hspace{1cm} (fluid 2, structure)
- \(<\text{fluid 1}, \text{fluid 2}>\): two-phase fluid-fluid Riemann problem

*Farhat, Rallu and Rajas, 2008*
- \(<\text{fluid 1, fluid 2}> : \text{two-phase fluid-fluid Riemann problem}\)
  
  \[- \rightarrow W_i^* \text{ and } W_j^* \]

\[\Phi_{ij}^{(1)} = \text{Roe}(W_i, W_i^*, \text{EOS}^{(1)}) \]

\[\Phi_{ij}^{(2)} = \text{Roe}(W_j, W_j^*, \text{EOS}^{(2)}) \]
Element deletion
- robust modeling of fracture
- widely used and understood by engineers and analysts
- no negative effect on time step
- no defined crack path
- loss of mass, momentum, and energy
EXTENDED FINITE ELEMENT METHOD

- XFEM with the Phantom Node Formulation*
  - each “cracked” element is replaced by two elements with phantom nodes
  - the cracking path within each element is tracked by a local signed distance function ($\phi(X, t)$)

$$I_c \triangleq \bigcup_{e \in \Omega_S} \{X \in e | \phi(X, t) = 0\}$$

* J-H Song et al. (2008)
Fluid-structure coupled computational framework

**AERO-F**
- FV compressible flow solver
- multi-material Riemann solvers
- level-set equation solver
- embedded boundary method

**DYNA**
- FE CSD solver
- fracture method
- XFEM
- element deletion

Embedded fluid-structure interface

**Experiment**

- implosive collapse of submerged aluminum tube (air-backed)
- increased water pressure until tube collapsed. Tube collapsed dynamically essentially under constant pressure (197.0 psi).

*Performed by S. Kyriakides et al. at University of Texas at Austin*
Simulation setup

- water / thin shell / air, no fracture
- modeled half of the tube length-wise (symmetry assumed)
- CFD grid: 3.7M nodes, 20.1M tetrahedron elements (300 procs.)
- structural model: 14K Belytschko-Tsay shell elements
- stress-strain response measured by experiment
- $J_2$-plasticity, piecewise linear hardening

UNDERWATER IMPLOSION

CFD domain

FE structural model (shell elements)

2D views
Synchronized Output from Experiment and Simulation
not available due to limited camera frequency

Probe A

Time: 0.000 ms

Probe B

Pressure (psi)

Time: 0.000 ms
not available
due to limited camera frequency

Time: 0.030 ms

Probe A

Probe B

Probe A

Probe B
not available due to limited camera frequency

Time: 0.110 ms

Probe A

Probe B

Probe A

Probe B
not available due to limited camera frequency

Time: 0.130 ms

Probe A

Pressure (psi)

Probe B

Time: 0.130 ms

Probe simulation

Probe experiment
Time: 0.150 ms

not available due to limited camera frequency

Pressure (psi)

Time (ms)

Probe A

Probe B

experiment
simulation

Eff. Plastic Strain

0.02
0.04
0.05

Probe A

Probe B

Time: 0.150 ms
not available due to limited camera frequency
Time: 0.326 ms

- **Probe A**
  - Experiment
  - Simulation

- **Probe B**
  - Experiment
  - Simulation
The image shows a comparison of experimental and simulation results for two probes, A and B, at a time of 0.576 ms. The graphs display the pressure (psi) over time (ms) for each probe, with green lines representing the experiment and red lines representing the simulation. The top left image shows the effective plastic strain with a color scale ranging from 0 to 0.05. The bottom left image provides a detailed view of the pressure distribution at the same time point.
The images show a comparison of experimental and simulation results. The top left image illustrates the effective plastic strain distribution. The bottom left image depicts the pressure distribution with two probes, labeled 'Probe A' and 'Probe B', at a time of 0.701 ms.

The graphs on the right side compare the pressure over time for 'Probe A' and 'Probe B'. The green line represents the experiment, while the red line represents the simulation. Both probes show a peak pressure at around 0.5 ms, with 'Probe A' having a slightly higher peak than 'Probe B' in the simulation.

The plots indicate that the simulation closely follows the experimental data, with minor discrepancies, suggesting good accuracy in the simulation model.
Experiment vs simulation for probe A and B, with pressure values shown in psi and time in ms.
Time: 1.201 ms
Time: 1.451 ms

Probe A

Probe B

Pressure (psi)

Time (ms)
**Validation**

![Graph showing pressure vs. time for experiment and simulation.](image)

**characteristics of the pressure pulse**

<table>
<thead>
<tr>
<th>Sensor No.</th>
<th>$p_{\text{min}}$ (psi)</th>
<th>$\Delta T^-$ (ms)</th>
<th>$I^-$ (psi-ms)</th>
<th>$p_{\text{max}}$ (psi)</th>
<th>$\Delta T^+$ (ms)</th>
<th>$I^+$ (psi-ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – exp.</td>
<td>-54</td>
<td>1.982</td>
<td>42.19</td>
<td>241</td>
<td>1.058</td>
<td>38.84</td>
</tr>
<tr>
<td>1 – sim.</td>
<td>-54</td>
<td>2.009</td>
<td>41.50</td>
<td>183</td>
<td>0.989</td>
<td>42.00</td>
</tr>
<tr>
<td>3 – exp.</td>
<td>-57</td>
<td>1.992</td>
<td>43.65</td>
<td>213</td>
<td>1.014</td>
<td>32.35</td>
</tr>
<tr>
<td>3 – sim.</td>
<td>-61</td>
<td>2.001</td>
<td>42.20</td>
<td>180</td>
<td>0.997</td>
<td>43.48</td>
</tr>
<tr>
<td>5 – exp.</td>
<td>-57</td>
<td>2.032</td>
<td>43.17</td>
<td>198</td>
<td>0.978</td>
<td>35.59</td>
</tr>
<tr>
<td>5 – sim.</td>
<td>-63</td>
<td>2.003</td>
<td>43.61</td>
<td>188</td>
<td>0.998</td>
<td>44.56</td>
</tr>
<tr>
<td>7 – exp.</td>
<td>-37</td>
<td>2.140</td>
<td>26.89</td>
<td>78</td>
<td>0.864</td>
<td>20.00</td>
</tr>
<tr>
<td>7 – sim.</td>
<td>-45</td>
<td>1.914</td>
<td>23.93</td>
<td>83</td>
<td>0.865</td>
<td>25.81</td>
</tr>
<tr>
<td>8 – exp.</td>
<td>-45</td>
<td>2.116</td>
<td>31.74</td>
<td>80</td>
<td>0.914</td>
<td>26.26</td>
</tr>
<tr>
<td>8 – sim.</td>
<td>-48</td>
<td>1.886</td>
<td>33.15</td>
<td>79</td>
<td>0.912</td>
<td>33.15</td>
</tr>
</tbody>
</table>

*Farhat, Wang, Kyriakides, et al. (2013)*
Underwater explosion and implosion
- tapered T6061-6 aluminum cylinder with 8 bulkheads
- blast loading (TNT detonation)
- fracture simulated by element deletion
Fracture of aluminum pipe driven by internal detonation.

*Performed by J. Shepherd et al. at California Institute of Technology.*
EXPERIMENTAL RESULT

- Crack propagation patterns

- **Direction of detonation wave**

- **Shot 120**: forward and backward curving, in the same direction

- **Shot 143**: forward and backward curving, in opposite directions

- **Shot 147**: forward and backward bifurcation
EXPERIMENTAL RESULT

- Blast pressure - notch length series

![Graph showing the relationship between blast pressure (kPa) and initial notch length (mm).]
Simulation setup

- detonation modeled by the Chapman-Jouguet theory
- three fluid materials: explosive gas \((C_2H_2 + O_2)\), detonation product, and water
- CFD grid: 1.4M nodes, 8.5M elements (on 100~200 proc. cores)
- CSD Model: 17K B-T shell elements; elasto-plastic
- initial notch: 25.4mm / 38.1mm / 50.8mm / 63.5mm / 76.2mm
Modeling the initial notch

- XFEM (2012): notch is modeled as an initial crack
- Element deletion (2014): notch is modeled by shells with reduced thickness
A 2D cut-view

2D Cut-view at y=0

Pressure (Pa)

Time: 0.000000

simulation with XFEM
Simulation with XFEM

Visualization of 3D result

Pressure Contours

Domain of Detonation Product
(Material Id: 2)

Time: 0.000000

Output Along Pipe Axis

Time: 0.000000
Structural deformation and stress

Simulation with XFEM
FRACTURE RESPONSE

- Comparison of XFEM and element deletion
  - XFEM: curving in the same direction or opposite directions
  - element deletion: branching
  - all these propagation patterns are observed in experiment

![XFEM, L= 25.4 mm](image1)

![XFEM, L= 38.1 mm](image2)

![ED, L= 38.1 mm](image3)
Peak blast pressure (L=38.1 mm)

- Experiment
- Simulation w/ XFEM: slope = 1.005 KPa/mm
- Simulation w/ elem. del.: slope = 0.906 KPa/mm
- Simulation w/ XFEM: slope = 0.614 KPa/mm
REFERENCES

- **Embedded boundary method**

- **Interface tracking**

- **Staggered time integrators**

- **FIVER**

- **Validation for underwater implosion**