DYNAMIC IMPLOSION OF UNDERWATER CYLINDRICAL SHELLS: Experiments and Computations

by

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Abstract

The implosion of an underwater structure is a dynamic event caused by the ambient constant pressure environment. It produces a short duration pressure pulse that radiates outwards and can damage adjacent structures. This paper presents results from a combined experimental/numerical study that aims to understand the underlying physics and establish the parameters that govern the nature of such pressure pulses. Collapse experiments on small-scale metal shells were conducted in a custom testing facility under constant pressure conditions representative of those in deep waters. The dynamic collapse of the shells was monitored using high-speed photography and the pressure around the structure with dynamic pressure transducers. Synchronization of the high-speed images with the data acquisition allowed temporal and spatial resolution of the events and the pressure pulses. Results from two experiments on shells that buckled and collapses in modes 4 and 2 are reported. A computational framework developed for the solution of highly nonlinear fluid-structure interaction problems characterized by shocks, large deformations, and self-contact is outlined. It features an Eulerian embedded boundary method for Computational Fluid Dynamics capable of achieving secondorder spatial accuracy including at the fluid-structure interface; an explicit structural analyzer with nonlinear geometric, material, and contact capabilities; and a loosely-coupled implicitexplicit fluid-structure time-integrator with a second-order time-accuracy and excellent numerical stability properties. The numerical tool is used to simulate the two experiments and shown to reproduce with good accuracy both the large deformations of the structure as well as the compression waves that emanate from it. The results demonstrate that the pressure pulse generated is influenced by the mode of buckling as well as the associated localization of collapse.

1. INTRODUCTION

The buckling of structures under external pressure is a classical stability problem that has received significant attention from the research community because it is a major design criterion for underwater structures (e.g., see Timoshenko and Gere [1961], Singer *et al.* [2002]). For metal structures such as cylindrical and spherical shells, buckling can start in the elastic regime but inelastic material behavior leads to catastrophic collapse. In a constant pressure environment like that in deep waters, the collapse is dynamic and the structure interacts with the surrounding fluid. The term *implosion*, as used here, refers to such a dynamic event. Although typically the designer of such structures is mainly concerned with avoidance of buckling and collapse, in some applications the dynamics of such implosion is of interest. In its simplest form, the fast inward traveling water surrounding the receding walls of a collapsing structure stops suddenly when the walls come into contact and reflect as shockwaves. Such pressure pulses can be large enough to damage adjacent structures and must be understood (Turner and Ambrico [2013]). The problem is further complicated if the collapsing structure fractures or fragments due to the additional interaction between the air inside the structure and the inward rushing water.

The significant amplitude, short duration and generally the signature of emanating pressure pulses were first recorded in experiments on glass spheres (e.g., Orr and Schoenberg [1976], Harben and Boro [2001], Turner [2007]), which under external pressure implode and shatter. The potential of destruction to neighboring structures was demonstrated in a 2001 accident that occurred at the Super-Kamiokande facility in Japan. The facility uses a large number of glass photomultiplier tubes in an underground tank filled with water to detect the mass of neutrinos. It was reported that during one filling of the tank one of the tubes imploded. The resultant shock damaged neighboring tubes causing them to implode; they in turn triggered their neighbors to implode and in this manner about 7,000 of the 11,000 tubes were destroyed in a short time leading to the closure of the facility for several months for expensive repairs (Cartlidge [2001]).

The influence of the pressurizing medium on the dynamic collapse of long tubes under external pressure has also been observed in the related problem of dynamic buckle propagation and arrest in offshore pipelines. In such long structures, collapse under external pressure starts locally but can propagate at velocities of the order of hundreds of m/s to quickly destroy essentially the whole structure. Experiments have shown that the velocity of buckle propagation

is significantly lower if the pressurizing medium is water rather than air (Kyriakides and Babcock [1979], Kyriakides and Netto [2000]). One way of limiting the extent of this catastrophic collapse is to periodically install ring stiffeners–buckle arrestors–at regular intervals along the line. The design of buckle arrestors requires understanding of the dynamics of the problem and the associated fluid-structure interaction (Netto and Kyriakides [2000a]). Dynamic propagation and arrest in air–or vacuum–have been simulated numerically successfully (Netto and Kyriakides [2000b]). Recognizing that compressibility of water plays a role in the radiated energy, the velocity of propagation has been calculated using an approximate fluid structure interaction model with some success (Song and Tassoulas, [1993], Netto *et al.* [1999]).

The aim of the present study has been: (a) to use experiments to establish the signature of pressure pulses emanating from imploding submerged metal structures; (b) to report on a numerical model that can simulate the transient high-speed fluid–structure interaction problem characterized by ultrahigh compressions, shock waves, large structural displacements and deformations, and self-contact; (c) to use the results to understand the factors that influence the signatures of such pressure pulses. This paper describes the progress made in characterizing and simulating the implosion problem. Section 2 briefly outlines the experimental facility developed for this study and then presents detailed results from two representative experiments (out of tens performed). Section 3 outlines the formulation behind the dynamic fluid and structural models developed and their interaction. Section 4 compares critically the results from simulations to those from the experiments, and Section 5 summarizes conclusions from this comparison.

2. EXPERIMENTAL STUDY

2.1. Implosion Experimental Set-up

Implosion is the dynamic collapse of an underwater structure or volume under constant pressure conditions. Part of the energy associated with the reduction of volume due to collapse is radiated out as a pressure wave. Of interest to this investigation is the signature of the emanating pressure pulse that is embodied in its peak amplitude, duration and impulse. Thus, the first objective of the investigation was to develop a custom pressure testing facility in which the constant pressure subsea conditions can be replicated, implosion events can be monitored, and the emanating pressure pulses recorded, free of interference from reflections. A photograph of the custom implosion facility is shown in Fig. 1a and in a schematic diagram in Fig. 1b. It consists of a 54-inch (1370 mm) internal diameter cylindrical pressure vessel with elliptical end-

caps. The vessel has an overall length of 102 in (3000 mm) and a seam-to-seam length of 72 in (1830 mm). These dimensions provide a reflection-free time window of approximately 1 ms. The facility has a pressure capacity of 1440 psi (100 bar) and can be pressurized by compressed air or water. It sits at a small inclination of 1.2° to the horizontal with an access hole in the top serving as a vent (see Fig. 1).

The vessel has an 18-inch (457 mm) diameter opening equipped with a Sentry quick opening door that allows operator access to the inside of the pressure vessel. A 6-inch (152 mm) opening on the opposite side is used for electrical signal access. Five 4-inch diameter windows arranged circumferentially at mid-length, as shown in the cross sectional view in Fig. 2, are used for lighting and observing the implosion events.

The facility was used to monitor the dynamic collapse of circular cylindrical shell specimens with diameters of 1.0-1.5 in (25-38 mm), various lengths and wall thicknesses. The shell is closed with solid steel plugs bonded in place as shown for example in Fig. 3a. It is then mounted onto a relatively unobtrusive adjustable cage-like frame in a way that allows free axial motion as shown in Fig. 4. The cage hangs in the vessel from thin wires that allow placement of the specimen in the vessel at the desired position—typically placed slightly below the axis of the vessel. The slim construction of the cage was chosen so as to minimize interference with the pressure pulse that emanates from the structure when it implodes.

The dynamic pressure associated with an implosion event is monitored with eight PCB Piezotronics-138A06 Underwater ICP Blast dynamic pressure sensors. In the experiments described here six of them were located in a circular arrangement at the mid-span of the specimen and two on the sides as shown in Fig. 4. The sensors were typically placed approximately one shell diameter from the surface of the shell, but the exact locations were recorded prior to each test.

The vessel is filled with water leaving a small air pocket at the top. It is pressurized with air using a pair of air boosters. Each specimen is designed to implode at a particular pressure. The pressure is gradually increased until the shell implodes. The test set-up provides a "soft" loading system that ensures that the far field pressure remains essentially constant during implosion—despite the small increase in the internal volume of the vessel caused by the collapse of the shell.

The dynamic pressure sensors consist of a tourmaline piezoelectric crystal enclosed in a soft polymeric tube filled with silicone oil. The crystal is connected to a signal conditioner (Dytran Instruments 4114B1) outside the vessel through a coaxial cable. Special hermetically sealed connectors were used to bring the cables outside the tank. The sensors have a resolution of 0.1 psi (0.69 kPa), a rise time of 1.5 µs, and a resonant frequency of 1 MHz.

The sensor signals are monitored via a National Instruments high-speed data acquisition system. The system consists of a PXI-1042Q chassis, an 8-channel PXI-6123 acquisition card, and a NI PXI-8196 PC using LabView. The system records at a rate of 500 kHz with a 16-bit resolution per channel. 8000 data points are acquired per channel over a total period of 16 ms. The system continuously records data, saving them when the signal from one of the sensors triggers it by reaching a predefined voltage. Data from the eight sensors are stored beginning at 8 ms before the trigger and ending at 8 ms after the trigger. The static pressure in the vessel is monitored by pressure gages and a pressure transducer connected to a separate data acquisition system.

The implosion event is monitored by one or two Photron Fastcam SA1 high-speed digital cameras looking through the windows as shown in Fig. 2. Zoom lenses enable viewing the entire length of the specimen for each case. The remaining windows are used to light the specimen with high intensity halogen lamps. To avoid overheating the PMMA windows, the lamps were turned on when the vessel pressure approached the calculated collapse pressure of the shell, and off soon after implosion. The cameras were run at 6,000-12,000 frames/s and images were captured at a resolution of about 1024×320 pixels storing events spanning up to 1.5 s.

As with the data acquisition system, the cameras are continuously capturing images but start to record them when triggered by a TTL signal from a pulse generator. The cameras are triggered by the same pressure signal as the data acquisition system. The trigger option was set at the center of the 1.5 s recording time, so that images pre- and post- the implosion event were recorded.

A unique feature of the testing facility is synchronization of the high-speed data acquisition system with the high-speed camera so that correspondence between the pressure signals and the images is achieved to within a few microseconds. This is accomplished by establishing a time relationship between the pressure and the video image time intervals based on the common trigger.

2.2. Experimental Results

a. L/D = 2.0

The first experiment to be analyzed involves a 1.5-inch diameter seamless Al-6061-T6 shell with a length-to-diameter ratio (L/D) of 2.0 and a diameter-to-thickness ratio (D/t) of 53.6 (see Table 1 for main dimensional and material parameters). The shell was sealed with solid steel plugs that penetrate it by 1.0 in (25 mm) as shown in Fig. 3a. The plugs have rounded ends and are bonded in place using epoxy, leaving the innermost 0.3 in (7.5 mm) unbonded.

Exp. No.	Tube No.	D in (mm)	t in (mm)	$\frac{L}{D}$	Δ ₀ (%)	Ξ ₀ (%)	E Msi (GPa)	σ _o ksi (MPa)	P _{CO} psi (bar)	Mode (\$°)
IMP69	A12	1.5007 (38.1)	0.0280	2.00	-	2.3	10.1 (69.6)	39.09 (270)	676 (44.6)	4 (37)
IMP88	A18	1.5019 (38.2)	0.0280 (0.711)	8.00	0.05	1.25	10.3 (70.8)	44.13 (304)	197 (13.6)	2 (0)

Table 1 Main geometric and material parameters of shells tested

The shell collapsed in mode 4 at a pressure of 676 psi (44.6 bar), a level that is close to the critical pressure predicted by elastic buckling shell analysis (e.g., Timoshenko and Gere [1962], Brush and Almroth [1975]). The event was captured with digital photography at a speed of 6,250 frames/s. Figure 5 shows a set of seven images of the dynamic collapse of the shell recorded by the LHS camera in Fig. 2. The locations of the crystals of the six central sensors are shown in Fig. 3b, which also shows schematically the orientation of the four extrados of the collapse mode. Figures 6a-6d show the dynamic pressure signals recorded from four of the sensors over a period of 3 ms. The times corresponding to the images are marked with numbered bullets on the four pressure signals.

A post-test photograph of the collapsed shell is shown in Fig. 7a. With the exception of a minor tear seen on the LHS of this photograph, the shell collapsed essentially intact (the tear is also seen on the top RHS of images (5) to (7) in Fig. 5). As is typical for drawn or extruded tubes, the tubular specimen exhibited a small amount of wall eccentricity $\Xi_o = 2.3\%$ defined by $\Xi_o = (t_{\text{max}} - t_{\text{min}})/(t_{\text{max}} + t_{\text{min}})$. This was sufficient to cause a small asymmetry in the collapse

mode with one side of the collapse penetrating more than the other three as illustrated in Fig. 7b. The small asymmetry resulted in some bending of the shell that can be seen to develop in images (to (in Fig. 5. In this case, the slightly thinner side of the shell was at the bottom and thus the bending occurred in the vertical plane of Fig. 5.

In image ① at T = -0.99 ms, the shell is intact and all pressure signals remain at zero. The walls start to recede at approximately -0.65 ms and simultaneously the dynamic sensors start recording a drop in the pressure. The receding walls can be seen in image ② at T = -0.35 ms. In image ③ at T = -0.19 ms, the walls continue to recede, the mode 4 collapse is clearly visible and the pressure continues to drop at all four locations. The drop in pressure with time differs to some degree between the four sensors because of differences in their locations relative to the four troughs that form. The minimum value of -124 psi (8.55 bar) is recorded by sensor 6 (the minimum pressures at the other locations are listed under P_{min} in Table 2—see definition schematic in Fig. 8). Beyond image ③, the inward motion of the collapsing walls starts to decelerate due to the built-up of axial tension and the inertial forces from the solid end-plugs. The deceleration leads to an increase in pressure so that by image ④ at -0.03 ms the pressure is near zero for all four sensors. The duration of the four negative pulses (ΔT^- in Fig. 8) given in Table 2 varies somewhat but is of the order of 0.80 ms. We define the pressure impulse, I^- , as the time integral of the pressure over ΔT^- and the values are listed in Table 2 (see also Fig. 8).

Sens. No.	P _{min} psi (bar)	$\Delta T^{-}_{\rm ms}$	<i>I</i> ⁻ psi.ms (bar.ms)	P _{max} psi (bar)	ΔT^+ ms	I ⁺ psi.ms (bar.ms)
1	-118	0.797	-36.90	472	0.206	37.78
-	(-8.14)	0.171	(2.54)	(32.55)	0.200	(2.61)
3	-94	0.767	-32.06	349	0.239	32.11
	(-6.48)		(2.21)	(24.07)		(2.21)
5	-113	0.815	-37.22	416	0.255	38.62
5	(-7.79)	0.015	(2.57)	(28.69)	0.233	(2.66)
6	-124	0.863	-40.35	622	0.218	39.41
	(-8.55)	0.005	(2.78)	(42.90)	0.210	(2.72)

Table 2 Characteristics of the pressure pulses for Exp. IMP69

We estimate that just past image 1 at -0.03 ms the walls of the collapsing shell start to come into contact, abruptly stopping the inward travel of water, which results in compression waves radiating outwards. These are captured by the four sensors as sharp positive pressure

pulses. The overall duration of the pressure pulses, listed in Table 2 under ΔT^+ , is of the order of 0.2-0.25 ms. The positive pressure pulses exhibit an initial high pressure spike followed shortly thereafter by a number of lower secondary ones most prominently displayed by sensor 1 in Fig. 6a. The higher pressure spikes were recorded by sensors 6 and 1, 622 psi and 472 psi respectively (42.9 and 32.6 bar) while sensors 3 and 5 recorded spikes of 349 and 416 psi (24.1 and 28.7 bar) respectively. By image ⑤ at T = 0.13 ms, the shell is fully collapsed and starts to bend slightly upwards and the recoded pressure is on the way down for all sensors. The pressure reaches zero level at all four locations by approximately T = 0.20 ms. A small negative depression develops followed by a small pressure peak at T = 0.29 ms which corresponds with image ⑥. The extent of the deformation of the shell remains the same but the overall bending has increased slightly. By image ⑦ at 0.45 ms, the implosion is fully completed and the pressure hovers about zero level for all sensors. The pressure disturbance between 0.88 and 1.15 ms is influenced by reflections from the tank wall. Image ⑦ shows the shell to have experienced slightly more overall bending.

The positive pressure impulses, I^+ , as defined in Fig. 8 are listed in Table 2. They vary to some degree with the one for sensor 6 being the largest. The variation is not so significant indicating that a nearly cylindrical pressure pulse is emanating from the imploding structure. Interestingly, although the positive pressure pulses have much shorter durations and much higher pressure peaks than the negative ones, the two impulses have very similar magnitudes at all locations.

b. L/D = 8.0

We now consider a shell with nearly the same diameter, wall thickness and basic material properties as the previous one but with an L/D of 8.0 (see Table 1). It was sealed with similar dimension steel plugs to those shown in Fig. 3a. The mid-length sensor arrangement is also similar (Fig. 9a) but two additional sensors were placed approximately at the quarter length positions on the lower surface of the shell as shown in Fig. 9b. Due to its much longer length the shell buckled and collapsed at a pressure of 197 psi (13.6 bar) in mode 2, again a level that corresponds quite well to elastic bifurcation buckling. The longer diameter of the mode 2 collapsing cross section is oriented along the 0-180° orientation as shown schematically in the inset of Fig. 9a. This corresponds to the orientation of the initial ovality reported in Table 1 as

 $\Delta_o = 0.05\%$. The dynamic implosion event was captured by the digital cameras at a frame rate of 8,000 fr/s, thus the images are separated by 0.125 ms. Figure 10 shows a set of nine images extracted from the recording of the LHS camera (see Fig. 2; here several images are separated by more than 0.125 ms). Figure 11 shows the signals recorded by four of the sensors over a period of ±1.5 ms on either side of the pulse. Once more, the times and pressures corresponding to the 10 images are marked with numbered bullets on the pressure signals.

Figure 12 shows two views of the collapsed specimen at the end of the experiment. Most of the length of the shell is significantly flattened with the cross section essentially folded (note the sharp curvatures of the edges of the cross section). The ends of the shell are deformed significantly due to constraints offered by the solid end-plugs.

Although the dynamic pressure recorded by all sensors at T = -1.5 ms is nearly zero, the complete photographic record shows that a small amount of ovalization started occurring at $T \approx -1.93$ ms. The walls begin to recede relatively slowly at first contributing no dynamic pressure. Their motion starts to accelerate approximately at -1.4 ms when the sensors start recording dynamic drops in pressure. By T = -0.674 ms, corresponding to image \oplus in Fig. 10, the shell is clearly ovalizing, the walls are receding, the pressure at sensor 1 is -31 psi (2.14 bar) and similar values are recorded by the other sensors at mid-span. The drop in pressure is initially gradual but by image 2, at -0.424 ms, the pressure at sensor 1 has dropped down to -49 psi (3.38 bar). The initially ovalized cross section has reverted to localized plastic deformation concentrated at four narrow axial zones at 90° angular intervals and the deformation is concentrated in a central section approximately 4D long (see image 3). Continuing with sensor 1, the pressure reaches a minimum of -54 psi (3.7 bar) just before ③ at T = -0.174 ms and records a very sharp spike (duration ~10 μ s) at T = 0. (Note that, given that the rise time of our sensors is 1.5 µs, the maximum pressure of this short duration spike may have been missed.) Such spikes are characteristic of mode 2 implosions and are presumably due to an initial essentially line contact of the collapsing walls of the shell near mid-span. Such spikes are usually used to trigger the data acquisition system and the cameras, as indeed was the case here. Interestingly, although sensor 5 is closer to one of the convex parts of the deforming shell cross section that is moving towards the sensor rather than away from it, the initial descending part of the signal is very similar to that of sensor 1 (see also Table 3). The first spike is of somewhat longer duration but has smaller amplitude. The remaining sensors at mid-span follow similar

trends to those seen in Fig. 11. Included in Table 3 are the minimum values of pressures recorded by five of the sensors (P_{\min}), the periods of the negative pressure parts of theirs signals (ΔT^{-}), and the impulse of these signals (I^{-}) (see definitions in Fig. 8).

Sens. No.	P _{min} psi (bar)	ΔT^{-} ms	I ⁻ psi.ms (bar.ms)	P _{max} psi (bar)	ΔT^+ ms	I ⁺ psi.ms (bar.ms)
1	-54 (-3.7)	1.982	42.19 (2.91)	241 (16.6)	1.058	38.84 (2.68)
3	-57 (3.9)	1.992	43.65 (3.01)	213 (14.7)	1.014	32.35 (2.23)
5	-57 (3.9)	2.032	43.17 (2.98)	198 (13.7)	0.978	35.94 (2.48)
7	-37 (-2.6)	2.140	26.89 (1.85)	78 (5.38)	0.864	20.00 (1.38)
8	-45 (-3.1)	2.116	31.74 (2.19)	80 (5.52)	0.914	26.26 (1.81)

Table 3 Characteristics of the pressure pulses for Exp. IMP88

Sensors 7 and 8 respectively located 3.28D and 2.36D on either side of mid-span recorded somewhat different pressure signals (Figs. 11c and 11d). Both sense the pressure trough that develops due to the receding walls. Sensor 7 recorded a smaller drop in pressure with the minimum value being -42 psi (2.9 bar) and a smaller I^- . Sensor 8 recorded a deeper trough (-52 psi—3.6 bar) and a higher I^- , presumably because it was located closer to the mid-section that is collapsing (see Fig. 9). The sharp pressure spike at T = 0 is missed by both indicating that this was a very local event with little radiated energy.

Returning to sensor 1, following the sharp spike at T = 0 the pressure returns back to negative values until about T = 0.068 ms when it enters positive territory. Simultaneously, contact of the walls at mid-span starts to spread as is evident from image ④ at T = 0.075 ms. Between image ④ and image ⑤ at 0.200 ms, the contacting area has grown covering a length of more than 2D. This rather sudden increase in the contact area implies that a wider column of fluid has come to a stop and is now rebounding as a compression wave. This in turn is responsible for the sharp rise in pressure to a peak of 241 psi (16.6 bar) at 0.180 ms and to the longer duration of this signal. At T = 0.214 ms, the pressure starts to drop and at 0.236 ms an undulating pressure plateau of 70 to 50 psi (4.83-3.45 bar) of 0.100 ms duration develops. Image ⑥ at 0.326 ms corresponds to the region close to the end of this plateau. During this time contact between the shell walls grows both longitudinally but also across the width. The spreading of contact continues in images \bigcirc and \circledast at 0.451 ms and 0.576 ms respectively; however the front of the collapse is progressively moving away from mid-span and consequently the pressures recorded by sensors 1 and 5 are decaying. Hence, at 0.451 ms the pressure at sensor 1 is about 34.5 psi (2.38 bar) and at 0.576 ms is 36 psi (2.48 bar). In image \circledast at 0.701 ms most of the shell has collapsed but a pressure of 12 psi (0.83 bar) is still registered by sensor 1. The pressure of this sensor drops to nearly zero at 0.87 ms somewhat past image \circledast . A last pressure peak is recorded with a maximum of 53 psi (3.65 bar) at 0.95 ms. This is caused by the collapsing shell engaging the solid end-plug, which terminates the deformation causing a compression wave to emanate from this area. The time it is recorded at mid-span has a built-in delay for the signal to travel the 6-inch distance that separates them.

The impulse of the signal of sensor 1 (I^+) reported in Table 3 includes the last pressure peak. It has a duration (ΔT^+) of 1.058 ms and a value of 38.84 psi.ms. Interestingly, it is somewhat smaller than I^- which, because of the localized nature of initial collapse, is due to the receding walls in the central half of the shell. By contrast, I^+ is from the signal measured at midspan resulting from contact of the walls first at mid-span but subsequently from a collapse front that is propagating outwards. Because of the length of the shell, the signal received at mid-span is somewhat attenuated due to the distance it has to travel.

The positive part of the pressure signal at sensor 5 in Fig. 11b has a slightly lower pressure peak—198 psi (13.7 bar) at 0.186 ms—as it is not facing the main area of collapse. The undulations that follow it are similar to those of sensor 1. A short plateau develops between points \bigcirc and \circledast followed by a decaying signal past point m. A smaller pressure peak with a maximum of 41 psi (2.83 bar) develops at essentially the same time as for sensor 1. Impulse I^+ listed in Table 3 is somewhat smaller than that of sensor 1.

The positive pressure signals of sensors 7 and 8 illustrate the attenuation of the pressure pulse with distance. Both sense the development of the major pulse at mid-span, but the maximum pressure recorded by 8 is 69 psi (4.76 bar) at 0.238 ms, and recorded by sensor 7 is only 78 psi (5.38 bar) at 0.250 ms. Furthermore, in image (5) at 0.201 ms the pressure signal has not arrived at sensor 7 and is just arriving at sensor 8 whereas for the central sensors the main pressure spike has developed and the pressure is starting to descend. The duration of this

pressure spike is also much shorter. Between times 0.326 ms (image [©]) and 0.826 (image [©]) the collapse is spreading and the sensor 8 pressure hovers between 15 and 25 psi (1.03-1.72 bar) and 8 to 16 psi (0.55-1.1 bar) for sensor 7. Both sensors record the pressure pulse that emanates from the compression waves that develop when collapse is finally arrested at the two end plugs. The signal at sensor 7 is a bit stronger than that of sensor 8 and arrives slightly earlier, again because sensor 7 is located closer to end-plug on the RHS (Fig. 9). Note that for both of these sensors the two end plug pulses are responsible for the maximum pressures (P_{max}) listed in Table 3. Needless to say that the impulses at these two locations listed in Table 3 are much smaller than those of the central sensors.

3. COMPUTATIONAL STUDY

As highlighted in the experimental study described above, the implosion of a submerged, gas-filled structure is a transient, high-speed, fluid-structure interaction problem characterized by ultrahigh compressions, shock waves, large structural displacements and deformations, and self-contact. Hence, the development of a computational approach for this problem is a formidable challenge. It requires incorporating in the computations material failure models, capturing the precise effects on the pressure peaks of many factors such as the rate of structural collapse, and accounting for the various interactions between the external fluid, the nonlinear structure, and the internal gas. Recently, a coupled fluid-structure computational framework that carefully addresses these challenges was presented in Farhat *et al.* [2012] and Wang *et al.* [2011, 2012]. This framework is adopted in this work and consequently is outlined in this section.

3.1 Computational Fluid Model

3.1a Equations of State

The underwater implosion process involves two fluids: water and air. It generates strong shock and expansion waves that propagate through both air and the surrounding water. This calls for modeling both media as compressible fluids.

Air typically behaves as a perfect gas. Therefore, it is represented here by the Equation of State (EOS) $P = (\gamma - 1)\rho e$, where *P*, ρ and *e* denote pressure, density and internal energy per unit mass, respectively, and γ denotes the ratio of specific heats.

Water can be assimilated with a stiffened gas whose EOS is $P = (\gamma_s - 1)\rho e - \gamma_s P_s$, where γ_s is an empirical constant, and P_s is a dimensional constant representing molecular attraction. For water and the system of imperial units, these two constants are experimentally determined to be $\gamma_s = 4.4$ and $P_s = 87,000$ psi (600 MPa).

Neglecting viscous effects, the dynamics of both air and water are modeled by the Euler equations.

3.1b Governing Equations of Dynamic Equilibrium

Let $\Omega_F \subset \Re^3$ denote the fluid domain of interest, that is, the union of the domains of water and air. The governing Euler equations can be written in vector and conservation form as

$$\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \boldsymbol{F}(\boldsymbol{W}) = 0 \quad \text{in } \ \boldsymbol{\Omega}_{F},\tag{1}$$

where *t* denotes here time,

$$\boldsymbol{W} = (\rho, \rho v_x, \rho v_y, \rho v_z, E)^T, \ \nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)^T, \text{ and } \boldsymbol{F}(\boldsymbol{W}) = \left(F_x(\boldsymbol{W}), F_y(\boldsymbol{W}), F_z(\boldsymbol{W})\right)^T, \quad (2)$$

$$F_x = \begin{cases} \rho v_x \\ P + \rho v_x^2 \\ \rho v_x v_y \\ \rho v_x v_y \\ \rho v_x v_z \\ v_x(E+P) \end{cases}, \quad F_y = \begin{cases} \rho v_y \\ \rho v_x v_y \\ P + \rho v_y^2 \\ \rho v_y v_z \\ v_y(E+P) \end{cases}, \quad F_z = \begin{cases} \rho v_z \\ \rho v_x v_y \\ \rho v_y v_z \\ P + \rho v_z^2 \\ v_z(E+P) \end{cases}, \quad (3)$$

 ρ denotes the fluid density, and *E* is its total energy per unit volume, *P* denotes the fluid pressure, and $\mathbf{v} = (v_x, v_y, v_z)^T$ is its velocity vector.

3.1c Non Body-Fitted Semi-Discretization

The large structural motions and deformations that characterize implosive collapse challenge the efficiency, if not robustness, of the mesh motion schemes (Farhat *et al.* [1998b], Degand and Farhat [2002]) needed for implementing an Arbitrary Lagrangian-Eulerian computational framework for fluid-structure interaction (Farhat *et al.* [1995, 2001], Lesoinne and Farhat [1998]). For this reason, the governing fluid Eq. (1) is kept here in its Eulerian setting and semi-discretized by the FIVER method (Finite Volume method with Exact two-phase Riemann

problems) described in Farhat *et al.* [2012a] on a *non body-fitted* grid. This allows handling large structural motions and deformations, contact, and even cracking, in a robust manner. The basic steps of FIVER are outlined below.

Let D_h denote a non body-fitted discretization of the fluid domain of interest Ω_F , where the subscript *h* designates the maximal length of the edges of this discretization. For every vertex $V_i \in D_h$, $i = 1, \dots, N_V$ a cell of control volume C_i is constructed. For example if D_h consists of tetrahedral, C_i is defined as the union of the volumes resulting from subdividing each tetrahedron D_h having V_i as a vertex by the quadrangular surfaces containing a mid-point of an edge of the tetrahedron, the centroid of this tetrahedron, and two of the face centroids (see Fig. 13). The boundary surface of C_i is denoted here by, ∂C_i and the unit outward normal to ∂C_i is denoted by $\mathbf{n}_i = (n_{i_x}, n_{i_y}, n_{i_z})^T$. The union of all of the control volumes defines a dual discretization of D_h .

Using the standard characteristic function associated with a control volume C_i , a standard variational approach, and integration by parts, Eq. (1) can be transformed into its weaker form

$$\int_{C_{i}} \frac{\partial W_{h}}{\partial t} d\Omega + \sum_{j \in K(i)} \int_{\partial C_{ij}} F(W_{h}) \cdot \mathbf{n}_{ij} d\Sigma + \int_{\partial C_{i} \cap \Sigma_{E}} F(W_{h}) \cdot \mathbf{n}_{E} d\Sigma + \int_{\partial C_{i} \cap \Sigma_{\infty}} F(W_{h}) \cdot \mathbf{n}_{\infty} d\Sigma = 0$$

$$< 1 > \qquad <2 > \qquad <3 > \quad (4)$$

where W_h denotes the semi-discrete state vector, K(i) denotes the set of neighboring vertices of V_i , $\partial C_{ij} = \partial C_i \cap \partial C_j$, n_{ij} is the unit outward normal to ∂C_{ij} , Σ_E is the discrete approximation of the wet surface of the structure—and therefore the fluid-structure interface— that is embedded in D_h , n_E is the unit outward normal to Σ_E , Σ_∞ denotes the far-field boundary of the flow, and n_∞ is its unit outward normal. This weaker form reveals that in practice, the flow computations are performed in a one-dimensional manner, essentially by evaluating fluxes along normal directions to boundaries of the control volumes. For this purpose, ∂C_i is split in the control volume boundary facets ∂C_{ij} connecting the centroids of the tetrahedra having V_i and V_j as common vertices. Then, term < 1 > in (4) is approximated by

$$\sum_{j \in K(i)} \int_{\partial C_{ij}} F(W_h) \cdot \boldsymbol{n}_{ij} d\Sigma \approx \sum_{j \in K(i)} \Phi_{Roe}(W_i, W_j, \text{EOS}, \boldsymbol{n}_{ij}),$$
(5)

where Φ_{Roe} is a numerical flux function associated with the second-order extension of the Roe flux [1981] based on the MUSCL (Monotonic Upwind Scheme Conservation Law) (van Leer [1979]), and W_i and W_j denote the average values of W_h in cells C_i and C_j , respectively. Term < 3 > in (4) is approximated by a far field boundary technique (Steger and Warming [1981], Ghidaglia and Pascal [2005]).The computation of term < 2 > involves a transmission condition that is introduced in Section 3.3a. This computation is discussed in Section 3.3b. Furthermore, in order to restore the correct asymptotic behavior of the pressure when the Mach number goes to zero in the regions of low-speed flow, the Roe flux is also equipped with local preconditioning (Turkel [1987]).

The ordinary differential equation resulting from the semi-discretization can be expressed in a compact form as:

$$\frac{d\mathbf{W}}{dt} + \mathbf{F}(\mathbf{W}) = 0,\tag{6}$$

where W and F denote the cell-averaged state vector and the numerical flux function for the entire mesh.

3.2 Computational Structural Model

3.2a Constitutive models

To properly account for geometric and material nonlinearities, the computational structural model is based on Green's second-order strain tensor and two different yield functions for modeling an elastic-plastic material:

(i) J_2 flow theory with isotropic hardening represented by

$$\sqrt{3J_2(s)} = \left(\frac{3}{2}s \cdot s\right)^{1/2} = \sigma_e \tag{7}$$

where s is the deviator of the second Piola-Kirchhoff stress tensor.

(ii) Barlat et al. [2003, 2004] anisotropic yield function represented by

$$|S_1' - S_2'|^8 + |2S_1'' + S_2''|^8 + |S_1'' + 2S_2''|^8 = 2\sigma_e^8,$$
(8)

where (S'_1, S'_2) and (S''_1, S''_2) are the principal values of the linearly transformed stress tensors S' and S'', respectively. These tensors are obtained from the stress deviator *s* and the stress tensor σ as follows:

$$S' = C's = C'T\sigma = L'\sigma$$
 and $S'' = C''s = C''T\sigma = L''\sigma$

where C', C'', T, L' and L'' are appropriate transformation tensors that allow introduction of the anisotropy (see Eqs. (4) of Korkolis and Kyriakides [2008], and Korkolis *et al.* [2010]).

3.2b Governing Equations of Motion

The Lagrangian equations of motion of the nonlinear flexible structure Ω_S can be written in compact form as

$$\rho_S \frac{\partial^2 u_j}{\partial t^2} = \frac{\partial}{\partial x_i} \left(\sigma_{ij} + \sigma_{im} \frac{\partial u_j}{\partial x_m} \right) + b_j \quad \text{in } \Omega_S, \quad j = 1, 2, 3, \tag{9}$$

where the subscripts designate the coordinate system (x, y, z), u is the displacement vector field of the structure, σ is the second Piola-Kirchhoff stress tensor, and b the vector of body forces acting in Ω_S .

3.2c Semi-Discretization

The nonlinear structural equations of motion (9) are typically semi-discretized by the Lagrangian finite element (FE) method. This leads to the discrete equations

$$M\frac{\partial^2 u}{\partial t^2} + f^{int}(u, \frac{\partial u}{\partial t}) = f^F(w) + f^{ext},$$
(10)

where M denotes the symmetric positive definite mass matrix, u denotes the vector of discrete structural displacements, and f^{int} , f^{ext} , and f^{F} denote the vectors of internal, external, and flow-induced discrete forces, respectively.

3.3 Fluid-Structure Interaction with Large Structural Deformations

3.3a Transmission Conditions

At the continuum level, the interaction between the fluid and structure subsystems represented by Eq. (1) and Eq. (9), respectively, is driven by two transmission conditions: the kinematic, non-penetration condition

$$\left(\boldsymbol{v} - \frac{\partial \boldsymbol{u}}{\partial t}\right) \cdot \boldsymbol{n}_{w} = 0 \quad \text{on} \ \boldsymbol{\Sigma}_{w},\tag{11}$$

and the equilibrium condition

$$\left(\sigma_{ij} + \sigma_{im}\frac{\partial u_j}{\partial x_m} + P\delta_{ij}\right)n_{w_i} - T_j = 0 \text{ on } \Sigma_w$$
(12)

where Σ_w denotes the physical fluid-structure interface whose FE discritization is Σ_E , n_w is the unit outward normal to Σ_w , and T_j denotes the tractions due to external forces whose origin is not due to the flow.

3.3b Embedded Computational Method for Fluid-Structure Interaction

The interface transmission conditions (11,12) are semi-discretized using the embedded boundary method for CFD (Computational Fluid Dynamics) described in (Wang *et al.* [2011,2012]) and consistent with FIVER. More specifically, the embedded discrete fluidstructure interface Σ_E is represented within D_h by the surrogate interface $\tilde{\Sigma}_E$ defined as

$$\tilde{\Sigma}_E = \bigcup_{(i,j)/V_i V_j \cap \Sigma_E \neq \emptyset} \partial C_{ij}$$
(13)

and constructed using either algorithm described in Wang *et al.* [2012]. Then, the interface condition (11) is enforced in two steps as follows:

(i) for each edge $V_i V_j$ of the fluid mesh that intersects the embedded discrete interface Σ_E that is, for each $V_i V_j \cap \Sigma_E \neq \emptyset$ (see Fig. 14), a one-dimensional fluid-structure Riemann problem between each vertex of this edge (V_i or V_j) and Σ_E , is constructed and solved using the structure velocity at the intersection point. (ii) for each edge $V_i V_j$ of the fluid mesh that intersects the embedded discrete interface Σ_E , a contribution to term $\langle 2 \rangle$ in Eq. (4) is evaluated by computing the numerical flux on each side of the fluid-structure interface as follows:

$$\int_{\partial C_{i} \cap \Sigma_{E}} F(W_{h}) \cdot \boldsymbol{n}_{E} d\Sigma \approx \int_{\partial C_{i} \cap \tilde{\Sigma}_{E}} F(W_{h}) \cdot \boldsymbol{n}_{ij} d\Sigma$$
$$= \int_{\partial C_{ij}} F(W_{h}^{(i)}) \cdot \boldsymbol{n}_{ij} d\Sigma \approx \Phi_{Roe}(W_{i}, W_{M}^{(i)}, EOS^{(i)}, \boldsymbol{n}_{ij})$$
(14)

and

$$\int_{\partial C_{j} \cap \Sigma_{E}} F(W_{h}) \cdot \boldsymbol{n}_{E} d\Sigma \approx \int_{\partial C_{j} \cap \tilde{\Sigma}_{E}} F(W_{h}) \cdot \boldsymbol{n}_{ji} d\Sigma = \int_{\partial C_{ij}} F(W_{h}^{(j)}) \cdot \boldsymbol{n}_{ji} d\Sigma$$
$$\approx \Phi_{Roe}(W_{j}, W_{M}^{(j)}, \text{EOS}^{(j)}, \boldsymbol{n}_{ji}), \tag{15}$$

where $W_M^{(i)}(W_M^{(j)})$ is the solution of the one-dimensional fluid-structure Riemann problem between $V_i(V_j)$ and Σ_E . It is notable that the velocity components of $W_M^{(i)}$ and $W_M^{(j)}$ verify the interface condition (11).

As for the interface condition (12), it is enforced using the conservative load transfer algorithm proposed in Wang *et al.* [2011] and based on work initially presented in Farhat *et al.* [1998a]. Briefly, this algorithm can be summarized as follows: (i) transform the pressure components of $W_M^{(i)}$ and $W_M^{(j)}$ into local force loads $f^F(P_M^{(i)})$ and $f^F(P_M^{(i)})$ and (ii) distribute these loads onto the wetted surface of the FE structural model as external forces.

3.3c Coupled Implicit-Explicit Time-Discetization

Finally, the semi-discrete fluid and structure subsystems are time-integrated by an Eulerian version of the second-order implicit-explicit staggered time-integrator described in Farhat *et al.* [2010]. In this coupled time-discretization algorithm, the semi-discrete fluid subsystem is time-integrated using the second-order three-point implicit backward difference formula and the semi-discrete structural subsystem is time-integrated using the second-order to deliver second-order time-accuracy and shown to possess excellent numerical stability properties thanks to its carefully designed coupling mechanisms.

4. SIMULATION RESULTS

The computational framework summarized above was implemented in the massively parallel AERO Suite of Codes (Farhat *et al.* [2003], Geuzaine *et al.* [2003]). It was also verified and validated for several large-scale, highly nonlinear applications associated with marine and aerospace engineering (e.g., see Wang *et al.* [2011, 2012]). It is applied here to simulate the two implosion experiments described in Section 2.

4.1 Simulation Methodology

In both cases, the origin of the Cartesian system adopted is at the geometric center of the specimen (see Figs. 15 and 20). For convenience, symmetry is assumed about the y - z plane. The shells are discretized using the Belytschko-Tsay four-node quadrilateral shell element (Belytschko *et al.* [1984]). Symmetry boundary conditions are applied to the nodes on the mid-span cross section. The ends of the shell are restrained radially, but are allowed to slide axially. Small initial geometric imperfections that correspond to the respective buckling modes are introduced to the shells via their mid-surface radii as follows:

$$R(\theta) = R_o(1 - a\cos n\theta), \tag{16}$$

thus n = 2 for IMP88 and n = 4 for IMP69.

For both experiments, the computational fluid domain is chosen to be an axis-aligned rectangular box with symmetry boundary at x = 0 and far-field boundaries on the other five faces. The non-reflecting far-field boundary conditions are enforced using the normal flux method (Ghidaglia and Pascal [2005]), which is particularly well suited for the EOS of water used in this work. This computational domain is discretized by an unstructured non-body-fitted tetrahedral mesh. Near the fluid-structure interface, the characteristic element size is of the order of the thickness of the shells. At time T = 0, the initial density, pressure, and velocity for air inside the specimen are set to $\rho_a^0 = 3.613 \times 10^{-5}$ lb/in³ (1 kg/m³), $P_a^0 = 14.5$ psi (1 bar), and $v_a^0 = 0$. The initial density and velocity of water outside the specimen are set to $\rho_w^0 = 0.03613$ lb/in³ (10³ kg/m³) and $v_w^0 = 0$. To imitate the pressure ramp-up process of the experiment, the water

pressure is initially set at $P_w^0 = P_{CO} + P_a^0 - 10$ psi and increased statically and linearly to $P_{CO} + P_a^0$, whereas the air pressure inside the specimen is maintained at P_a^0 . Only when the collapse pressure is reached, the time-integration of the fluid subsystem is started. The duration of the pressure increase process, ΔT_{inc} , is chosen such that when $P_{CO} + P_a^0$ is reached, the structural model has just started collapsing.

The time-step is chosen to be the same for both semi-discrete fluid and structural subsystems to avoid sub-cycling and maximize time-accuracy. This time-step is maintained constant (65 ns for IMP69 and 40 ns for IMP88) through the entire simulation. In each case, it corresponds to a large fluid CFL (Courant–Friedrichs–Lewy) number, which maximizes computational efficiency. The simulation is terminated after the specimen has fully collapsed and the pressure disturbance in water gets close to zero.

4.2 L/D = 2.0 (IMP69)

A schematic drawing of the structure and non-body-fitted fluid mesh (truncated view) is shown in Fig. 15. The structure consists of the shell and the solid plug. The shell is discretized with 53 elements along the half-length and 136 elements around the circumference. The steel plug including its rounded end is discretized using 38,920 four-node elastic tetrahedral elements. The plug and the shell share the same nodes along the circumference at point A. The shell material is modeled as a bilinear elastic-plastic solid with an elastic modulus of 10.1 Msi (69.6 GPa), yield stress of 42.4 ksi (292 MPa), post-yield modulus of 97.8 ksi (674 MPa), Poisson ratio equal to 0.3 and density $\rho^s = 0.10$ lb/in³ (2779 kg/m³). The computational fluid domain is chosen to be $\Omega = \{x, y, z \in \mathbb{R}^3 : 0 \le x \le 12 \text{ in}, -10 \le y, z \le 10 \text{ in}\}$. This domain is discretized with 4,908,508 nodes and 29,298,618 tetrahedra. The element size near the fluid-structure interface is ~0.03 in (0.76 mm) and progressively coarsens to about 3 in (76 mm) at the outer boundaries (not shown in Fig. 15).

The simulation is started with a pressure increase process that has a duration of $T_{inc} = 0.476$ ms, and is terminated at $T_{final} = 1.65$ ms. The shell buckles and collapses dynamically in mode 4, mimicking the experiment. Figure 16 shows eight images of the shell at different levels of deformation (for visualization purposes the half shell model is reflected about the plane of symmetry). The pressure time-histories predicted at two sensor locations are shown in Fig. 17,

together with the corresponding experimental signals. (Note that sensor locations are mapped onto the computational domain by replicating the orientation of the collapse mode.) In order to facilitate comparison between the computed and measured results, the time origin is shifted so that the highest pressure peak at sensor 1 is aligned with that of the experiment. Marked on the calculated response with numbered bullets are the locations that correspond to the images in Fig. 16. Color 3D rendering of the pressure fields around the collapsing shell at six instances that again correspond to the labeled points on the response appear in Fig. 18.

The reader can observe that the main features of the experimental signals are captured quite well by the simulation. These include the initial relatively gradual pressure drop, followed by a sharp pressure pulse with multiple spikes, and ending by a shallow pressure drop. Image [1] is taken at the beginning of the simulation (T = -1.214 ms) when the pressurization step starts. At this time the structure is intact and the fluid is at rest. The flow solver is activated at -0.725 ms and leads to the small pressure oscillations seen in the signals of the two sensors in Fig. 17. At this time the structure has slightly deformed. Its wetted surface has a non-zero velocity, whereas the fluids (both water and air) still have zero velocity everywhere. Around T = -0.70 ms, the shell starts to deform dynamically in mode 4. Deformation localizes around mid-span as illustrated in image [2] at -0.364 ms in Fig. 16. Simultaneously, the calculated pressure drops because of the receding walls of the shell.

Sens.	P _{min} psi	ΔT^{-}	<i>I</i> [−] psi-ms	P _{max} psi	ΔT^+	I ⁺ psi-ms
No.	(bar)	ms	(bar-ms)	(bar)	ms	(bar-ms)
1	-130	0.604	41.15	409	0.323	30.46
	(-8.97)	0.074	(2.84)	(28.2)		(2.10)
3	-115	0.697	36.02	360	0.320	25.34
	(-7.93)		(2.48)	(24.8)		(1.75)
5	-126	0.679	39.92	339	0.317	28.52
5	(-8.69)	0.077	(2.75)	(23.4)	0.517	(1.97)
6	-132	0.686	41.38	357	0.314	30.17
	(-9.10)		(2.85)	(24.6)		(2.08)

Table 4 Characteristics of calculated pressure pulses for IMP69

In image [3] at -0.174 ms, the four developing troughs have grown while the pressure reaches its lowest value. Listed in Table 4 are the minimum values recorded at the locations of four of the sensors. Although they do not exactly match the experimental values in Table 2, on average they are within 12.5% of these values. In image [4] at T = -0.044 ms, the inward deformation has

grown further and the walls are close to first contact. The pressure starts to rise and turns positive at approximately T = -0.033 ms. As in the experiment, this time is defined as the end of the negative pulse. The duration and impulse of this negative pulse are listed in Table 4 for sensors 1, 3, 5 and 6, and compared with the experimental data shown in Table 2. The agreement for I^{-} is good with an average difference of 8.75%. Comparison of the duration is somewhat inappropriate due to the way the dynamic calculation is commenced.

Figure 19 shows the shell at first contact, which occurs at T = -0.006 ms. Contact is limited to the plane of symmetry where the deepest parts of the cross-shaped intrados are located (note that the figure shows the shell mid-surface whereas contact occurs at the inner surface of the shell; in other words, the gap represents one wall thickness). Interestingly, just before first contact, the shell wall velocity is somewhat above 3000 in/s (76 m/s). Furthermore, at first contact the water pressure on the surface of the shell is 8,444 psi (582 bar). This in fact is the highest pressure recorded in the entire calculation.

After first contact, the contact region expands rapidly in the axial direction accompanied by significant flattening. Simultaneously, a strong positive pressure pulse emanates and propagates outward in the water as illustrated in image [5] of Fig. 18 at T = 0.026 ms. The signal is strongest on the surface of the shell and decays as it radiates outwards. By this time, roughly one-half of the shell length has flattened while the two sensors are recording a sharp rise in pressure. As in the experiment, sensor 1 records the highest pressure of 409 psi (28.2 bar) at T =0.033 ms, which is 13% lower than the measured value. It is interesting to confirm that this maximum corresponds to first contact; in other words, the time delay is related to the time of travel of the shock from the surface of the shell to the location of the sensor (see Fig. 3).

The maximum pressures at four of the sensor locations are reported in Table 4. For sensors 1, 3 and 5, the values are in good agreement with their experimental counterparts shown in Table 2, with differences ranging from 3% to 22%. By contrast, at sensor 6 the maximum pressure is about 43% lower than in the experiment. In view of the assumed symmetry about the mid-span and that the wall eccentricity was not included in the model these comparisons are deemed to be quite favorable.

Following the pressure maximum, the water pressure starts to decrease as the first wave of shocks moves past the sensors. However, around the time of image [6] (T = 0.082 ms), the collapse has reached the solid end-plugs, and the shell bends over its rounded shape. This

abruptly stops the forward motion of the collapse and generates a compressive shock wave that can be seen in image [6] in Fig. 18. This shock is also recorded by the two sensors in Fig. 17 as secondary spikes. The maximum value of this spike is found to be 289 psi (19.9 bar) at sensor 1 and 255 psi (17.6 bar) at sensor 3.

After the second spike, the pressure drops sharply as the shell has collapsed completely. In image [7] at 0.136 ms, the water pressure near the shell has turned negative. Correspondingly, the pressure at sensors 1 and 3 reaches zero at 0.142 ms and continues to drop. The duration and impulse of the positive pressure pulse are reported in Table 4. Overall the values of I^+ are somewhat lower than those in the experiment.

In the analysis, a small negative pressure pulse is recorded at all sensor locations between images [7] and [8] at 0.286 ms. Although several of the sensors in the experiment exhibited a pressure drop at about the same time, the drops are shallower with smaller impulse. We suspect that, once more, small differences in the deformation of the shell at this time frame, caused by asymmetries in the experimental shell are responsible for this difference. The pressure at all sensors positions returns to zero at T = 0.25 ms and hovers about the zero level afterwards. The simulation was terminated shortly after image [8] at 0.286 ms.

4.3 L/D = 8.0 (IMP88)

The geometric parameters of this specimen are listed in Table 1. Symmetry about midspan is again assumed, thereby reducing the solid and fluid meshes to those shown in Fig. 20 (the view of the water domain is truncated). The shell is discretized with 140 elements along the halflength and 100 elements around the circumference. The steel plug is represented by 50 four-node rigid elements with the correct total mass. The shell material is modeled as an elastic-plastic solid with the non-quadratic anisotropic yield function used in Korkolis and Kyriakides [2008] and calibrated for this material in Korkolis *et al.* [2010]. A piecewise linear fit of the measured tensile stress-strain response is adopted. Its elastic modulus, Poisson ratio, and density are respectively set to 10.3 Msi (70.8 GPa), 0.3, and 0.10 lb/in³ (2779 kg/m³). The computational fluid domain is chosen as $\Omega = \{x, y, z \in \Re^3 : 0 \le x \le 100 \text{ in}, -75 \le y, z \le 75 \text{ in}\}$. It is discretized with 3,377,863 nodes and 20,128,823 tetrahedra. The element size near the fluid-structure interface is ~0.03 in (0.76 mm) and progressively coarsens to about 10 in (254 mm) at the outer boundaries (not shown in Fig. 20). The simulation began with static pressure increase for a duration of $T_{inc} = 0.3$ ms, and was terminated at $T_{final} = 5.5$ ms. As expected, the shell collapses dynamically in mode 2, mimicking the experiment. Figure 21 shows six images of the shell at different levels of deformation. Again, reflection is applied to obtain the results for the full specimen. The pressuretime-histories predicted at three sensor locations (including one off the symmetry plane) are plotted in Fig. 22 together with the corresponding measured responses. As in the case of IMP69, the time origin of the simulation results is shifted to match the main events of the experiment. The six images in Fig. 21 correspond to the points marked on the calculated response with numbered solid bullets. Three-dimensional renderings of the pressure field in the neighborhood of the shell at the same six instances are shown in Fig. 23.

Comparing the calculated results in this set of figures with the experimental results shown in Figs. 10-12, it is clear that the simulation captures the main features of this implosion event quite faithfully. In the simulation, the shell wall begins to recede slowly near the end of the pressurization step. The dynamic fluid solver is activated at T = -2.64 ms. Due to the discontinuity of the velocity across the fluid-structure interface, a small oscillation develops in the fluid pressure field at this time, as in the simulation of IMP69. This oscillation dies out rapidly and at T = -1.5 ms—when the pressure plots in Fig. 22 start—it is no longer noticeable. At this time, the inward motion of the shell walls accelerates and from T = -1.4 ms onwards this causes the dynamic pressure to start dropping at all three sensor locations shown. Image [1] in Fig. 21 at -0.23 ms, shows the shell deformed in mode 2, and the deformation localized in the central half of the model. The pressure at sensors 1 and 5, on the symmetry plane, has reached the minimum, whereas at sensor 8, about 2.4D away, it continues to drop. The descending parts of the three responses match the corresponding experimental ones very well. The minimum pressure at five sensor locations listed in Table 5 are seen to be close to the corresponding experimental values (Table 3) with an average difference of only 5.0%.

The collapsing walls of the shell model at mid-span come into first contact at approximately T = -0.055 ms. The local cross section has deformed to the characteristic "8" shape shown in Fig. 24a. Because of the axial localization of the collapse, contact is along a line formed by the intrados of the deformed cross section along a short axial length of about ~0.5D—note once again that Fig. 24 shows the mid-surface of the shell whereas contact occurs on the inner surface at the half wall thickness. The contact points, which had an inward velocity of 872

in/s (22.1 m/s), abruptly come to a stop generating a wave of outgoing shocks in the surrounding water. This wave is responsible for the first narrow pressure spike recorded at sensor positions 1 and 5 (see Fig. 22a and 22b). Because contact is limited to a narrow axial zone, the pressure returns quickly to negative values.

Sens.	P _{min} psi	ΔT^{-}	<i>I</i> ⁻psi-ms	P _{max} psi	ΔT^+	I ⁺ psi-ms
No.	(bar)	ms	(bar-ms)	(bar)	ms	(bar-ms)
1	-54	2 009	41.50	183	0 989	42.00
1	(3.72)	2.007	(2.86)	(12.62)	0.767	(2.90)
3	-61	2 001	42.20	180	0 997	43.48
5	(4.21)	2.001	(2.91)	(12.41)	0.777	(3.00)
5	-63	2 003	43.61	188	0 998	44.56
5	(4.34)	2.005	(3.01)	(12.97)	0.770	(3.07)
7	-45	1 01/	23.93	83	0.865	25.81
/	(3.10)	1.714	(1.65)	(5.72)	0.005	(1.78)
8	-48	1 886	33.15	79	0.912	33.15
0	(3.31)	1.000	(2.29)	(5.45)	0.712	(2.29)

Table 5 Characteristics of calculated pressure pulses for IMP88

The mid-span cross section continues to collapse, and contact quickly expands both transversely and axially as shown in Fig. 24b. This corresponds to the time of a second narrow pressure spike of higher amplitude at about 0.03 ms. By the time it is over, approximately a contact length of 0.4D has developed across the most deformed cross section, and the axial length has grown to 1.25D. Soon thereafter, the mid-span cross section is essentially flattened, and contact has spread axially to about 2D as illustrated in image Fig. 24c at 0.120 ms. This corresponds to the main spike recorded by the sensors. The mid-span sensors record their highest values at 0.150 ms. By configuration ③ at 0.165 ms, the pressure is dropping at the central sensor locations even though, as shown in Fig. 21 only the central 1/3 of the shell has come into self-contact. This supports the experimental observation that because collapse localizes, the pressure spike of main concern is due to first contact when the local velocity of the collapsing shell is the highest (consistent with results in Turner and Ambrico [2013]).

The pressure maxima are listed under P_{max} in Table 5. Comparing them with those of the corresponding experimental sensors listed in Table 3, they can be declared to be in reasonable agreement (average difference ~16%). Interestingly, as in the experiment, sensor 8

1D away records both the first short spike as well as the second longer one with some delay and at reduced amplitudes.

Pressure at all sensor locations starts to decrease after the major spike moves past these sensors. Images [4] at T = 0.274 ms and [5] at T = 0.565 ms are taken while collapse and contact spread axially towards the steel plugs. The full pressure fields corresponding to these two images (Fig. 23) show a local pressure pulse emanating from the propagating collapse front. Concurrently, the pressure at sensors 1 and 3 continues to drop as the collapse front moves away from them. By contrast, the pressure at sensor 8 gradually increases. At approximately the time of image [6] (T = 0.850 ms), the collapse front has reached the end-plug and is abruptly stopped. This generates a relatively strong wave of shocks, captured at all the sensor locations as a relatively long pressure pulse, as in the experimental records. In fact, this last pressure pulse is responsible for the highest pressure recorded at sensor 8 (Table 5), which matches well the experimental result (Table 2). After this last wave of shocks, the pressure decreases at all sensor locations. It turns negative at about T = 1.2 ms, and hovers around the zero level afterwards. The shell is completely collapsed. Figure 25, shows contours of equivalent plastic strain superimposed on the final collapsed configuration of the shell. The highest levels of strain occur at the interface with the solid plug where often such shells failed in the experiments.

5. SUMMARY AND CONCLUSIONS

The implosion of an underwater structure is a dynamic event caused by the ambient constant pressure environment. Initially, the surrounding water follows the dynamically receding walls of the collapsing structure but comes suddenly to a stop when the walls of the structure come into contact. This produces a short duration pressure pulse that radiates outwards. Such a pressure pulse can have sufficient amplitude and impulse to cause damage to a neighboring vessel and must the understood. The aim of this study was to first use experiments to quantify the signature of such pressure pulses and understand the factors that govern them; second to develop a modeling framework capable of capturing the complex fluid-structure interaction associated with such implosion events; and third to evaluate the performance of the models developed and use the results to understand the factors that govern the signatures of such pressure pulses. This paper summarized the results of these efforts.

The experimental study involved the development of a custom testing facility that allows for the dynamic collapse of small-scale shells under conditions that simulate the constant pressure underwater environment. The dynamic implosion process, which has a time scale of the order of one millisecond, is monitored with high-speed digital photography. The emanating pressure pulse is measured with dynamic pressure sensors that surround the specimen and recorded on a high-speed data acquisition system. A unique feature of the testing facility is the synchronization of the high-speed images with the recorded pressure pulses that allows temporal and spatial resolution of the events and the pressure pulses.

The paper presented representative results from two implosion experiments on aluminum cylindrical shells with a D/t of about 54: one with an L/D = 2 that collapsed in mode 4 and a second with an L/D = 8 that did so in mode 2. In both cases the structures buckled elastically at the expected pressure levels and subsequently collapsed dynamically due to additional loss of stiffness resulting from inelastic action. The synchronized photographic image and pressure sensor records showed that during the initial inward motion of the shell walls, the local pressure drops below the ambient level. A sharp short duration spike was recorded during first contact of the shell walls followed by a longer and much higher amplitude pressure pulse that developed when a more significant part of the shell came into contact. In the case of the shorter shell, the pressure pulse was influenced by the deformation of the whole structure. Furthermore, the impulse of the initial drop in pressure was approximately equal to that of the shorter but higher amplitude compression signal that followed. In the case of the longer shell, collapse localized in a length of about 4D and this was responsible for the main part of the compression pressure pulse. Collapse subsequently propagated to the ends of the shell but this yielded rather lower amplitude pressure signals. In both cases the structures did not fracture in any significant manner. The duration and signature of the outgoing pulse is influenced by the mode of collapse with mode 4 producing a higher amplitude but shorter duration pressure pulse than mode 2.

A comprehensive computational framework developed in the course of this study for the solution of highly nonlinear fluid-structure interaction problems characterized by shocks, large deformations, and self-contact reported elsewhere has been outlined. It features an Eulerian embedded boundary method for Computational Fluid Dynamics capable of achieving second-order spatial accuracy including at the fluid-structure interface, an explicit structural analyzer with nonlinear geometric, material, and contact capabilities, and a state-of-the-art loosely-

coupled implicit-explicit fluid-structure time-integrator with provable second-order timeaccuracy and excellent numerical stability properties.

The performance of the numerical simulation technology developed was evaluated by direct comparison of simulation results with the two experiments described in Section 2. It was found that it always delivered clean and stable time-histories. The initial pressure drops arising from the implosion process were reproduced quite well and the pressure spikes that follow reasonably well. In the case of IMP69, deviations from the experimental results are believed to be due to the wall eccentricity present in the shell tested but not accounted for in the models. Indeed, the computed results agree better with their experimental counterparts for experiment IMP88 where geometric imperfections in the cylindrical shell played a lesser role. It was also found that in both cases, the numerical tool reproduces very well the time instances at which the pressure peaks and drops occur, and the pressure impulse. Furthermore, visual comparisons of the structural deformations including the extent and evolution of contact, and the local curvatures of the collapsed shells showed the computed collapsed shapes to agree well with their experimental counterparts.

The numerical tool provided additional insight into the events associated with the two implosions. It was confirmed that the highest pressure occurs at the instant of first contact and that the local inward velocity of the through formed by localized collapse is the highest just before contact. Furthermore, first contact is limited to a very short section of the collapsing walls. By contrast, the major pressure spike comes a short time later when a somewhat larger section of the shell walls come into contact. This is more prominently displayed in the case of the longer shell where the main pressure spike occurs when only a relatively short section of the shell has collapsed. This is because collapse is localized and only a short section develops the high velocity responsible for high compression shock after it comes to a stop by contact. In the subsequent propagation of the front, the velocities at contact are lower and thus the emanating pressure signals are much lower. In view of these events, in both cases the high pressure signals propagated nearly in a spherical manner and their amplitude decays as r^{-2} at distances beyond one radius from the shell surface.

The study included a significant number of experiments in which the problem parameters such as the shell D/t, L/D, material, and boundary conditions were varied. Results and

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observations on the effect on these on the emanating pressure pulses will follow in future publications.

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REFERENCES

- Barlat, F., Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Lege, D.J., Pourboghrat, F., Choi, S.-H. and Chu, E., 2003. Plane stress yield function for aluminum alloy sheets-Part I: Theory. *Int'l J. Plasticity* 19, 1297-1319.
- Belytschko, T., Lin J., Tsay, C., 1984. Explicit algorithms for the nonlinear dynamics of shells. *Comp. Meth. Appl. Mech. Eng.* **42**, 225–251.
- Brush, D.O and Almroth, B.O., 1975. Buckling of Bars, Plates and Shells. McGraw-Hill.
- Cartlidge, E., 2001. Accident grounds nutrino lab. Physicsworld.com, Nov. 15, 2001.
- Degand, C., Farhat, C., 2002. A three-dimensional torsional spring analogy method for unstructured dynamic meshes. *Comput. Struct.* **80**, 305–316.
- Dyau, J.Y. and Kyriakides, S., 1993. On the localization and collapse in cylindrical shells under external pressure. *Int'l J. Solids Struct.* **30**, 463-482, 1993.
- Farhat, C., Lesoinne, M. and Maman, N., 1995. Mixed explicit/implicit time integration of coupled aeroelastic problems: Three-field formulation, geometric conservation and distributed solution. *Int'l J. Num. Meth. Fluids* 21, 807-835.
- Farhat, C., Lesoinne, M. and LeTallec, P., 1998a. Load and motion transfer algorithms for fluid/structure interaction problems with non-matching discrete interfaces: momentum and energy conservation, optimal discretization and application to aeroelasticity, *Comp. Meth. Appl. Mech. Eng.* 157, 95-114.
- Farhat, C., Degand, C., Koobus, B., Lesoinne, M., 1998b. Torsional springs for two-dimensional dynamic unstructured fluid meshes. *Comp. Meth. Appl. Mech. Eng.* **163**, 231–245.

- Farhat, C., Geuzaine, P., Brown G., 2003. Application of a three-field nonlinear fluid-structure formulation to the prediction of the aeroelastic parameters of an F-16 fighter. *Comput. Fluids* 32, 3–29.
- Farhat, C., Geuzaine, P., Grandmont, C., 2001. The discrete geometric conservation law and the nonlinear stability of ALE schemes for the solution of flow problems on moving grids. J. Comput. Physics 174, 669–694.
- Farhat, C., Rallu, A., Wang, K., and Belytschko, T., 2010. Robust and provably second-order explicit-explicit and implicit-explicit staggered time-integrators for highly non-linear compressible fluid-structure interaction problems. *Int'l J. Num. Methods Eng.* 84, 73-137.
- Farhat, C., Gerbeau, J.-F., Rallu, A., 2012. FIVER: A finite volume method based on exact twophase Riemann problems and sparse grids for multi-material flows with large density jumps. *J. Comput. Physics* 231, 6360-6379.
- Geuzaine, P., Brown, G., Harris, C. and Farhat, C., 2003. Aeroelastic dynamic analysis of a full F-16 configuration for various flight conditions. *AIAA J.* **41**, 363-371.
- Ghidaglia, J.-M. and Pascal, F., 2005. The normal flux boundary method at the boundary for multidimensional finite volume approximations in CFD. *Europ. J. Mech. B/Fluids* **24**,1–17.
- Harben, P. and Boro, C., 2001. Implosion source development and Diego Garcia reflections. 23rd Seismic Research Review: Worldwide Monitoring of Nuclear Explosions, Oct. 2-5, pp., 23-31.
- Korkolis, Y.P. and Kyriakides, S., 2008. Inflation and burst of anisotropic aluminum tubes Part II: An advanced yield function including deformation-induced anisotropy. *Int'l J. Plasticity* 24, pp. 1625-1637.
- Korkolis, Y.P., Kyriakides, S., Giamouris, T., Lee, L.-H., 2010. Constitutive modeling and rupture predictions of AL-6061-T6 tubes under biaxial loads. ASME J. Appl. Mech. 77, 064501 1-5.
- Kyriakides, S. and Babcock, C.D., 1979. On the dynamics and the arrest of the propagating buckle in offshore pipelines. Proc. Offshore Technology Conference, Houston TX, OTC Paper 3479, Vol. II, pp. 1035-1045.
- Kyriakides, S. and Netto, T.A., 2000a. On the dynamics of propagating buckles in pipelines. *Int'l J. Solids Struct.* **37**, 6843-6867.
- Lesoinne, M. and Farhat, C., 1998. A higher-order subiteration free staggered algorithm for nonlinear transient aeroelastic problems. *AIAA J.* **36**, 1754-1756.
- Netto, T.A. and Kyriakides, S., 2000a. Dynamic performance of integral buckle arrestors for offshore pipelines. Part I experiments. *Int'l J. Mech. Sci.* **42**, 1405-1423.

- Netto, T.A. and Kyriakides, S., 2000b. Dynamic performance of integral buckle arrestors for offshore pipelines. Part II analysis. *Int'l J. Mech. Sci.* **42**, 1425-1452.
- Netto, T.A., Kyriakides, S. and Ouyang, X., 1999. On the initiation and propagation of buckles on a beam on a nonlinear foundation. *ASME J. Applied Mechanics* **66**, 418-426.
- Orr, M., Schoenberg, M., 1976. Acoustic signatures from deep water implosions of spherical cavities. J. Acoustical Soc. America 59, 1155-1159.
- Roe, P.L., 1981. Approximate Riemann solvers, parameters vectors and difference schemes. J. Comput. Physics, 43, 357–371.
- Singer, J., Arbocz, J., Weller, T., 2001. Buckling Experiments: Experimental Methods in Buckling of Thin-Walled Structures, Vol. 2, John Wiley & Sons, New York.
- Song, H.-W., Tassoulas, J.L., 1993. Finite element analysis of propagating buckles. *Int'l J. Num. Meth. Eng.* **36**, 3529-3552.
- Steger, J.L., Warming, R.F., 1981. Flux vector splitting of the inviscid gas dynamic equations with application to finite-difference methods. *J. Comput. Physics* **40**, 263–293.
- Timoshenko, S.P. and Gere, J.M., 1961. Theory of Elastic Stability 2nd Edition. McGraw-Hill.
- Turner, S.E., Ambrico, J.M., 2013. Underwater implosion of cylindrical metal tubes. *ASME J. Appl. Mech.* **80**, 011013-1-11.
- Turner, S.E., 2007. Underwater implosion of glass spheres. J. Acoustical Soc. Amer. 121, 844-852.
- Turkel, E., 1987. Preconditioned methods for solving the incompressible and low-speed compressible equations. J. Comput. Physics 72, 277–298.
- Van Leer, B., 1979. Towards the ultimate conservative difference scheme V: a second-order sequel to Godonov's method. *J. Comput. Physics* **32**, 361–370.
- Wang, K., Rallu, A., Gerbeau J.-F., Farhat C., 2011. Algorithms for interface treatment and load computation in embedded boundary methods for fluid and fluid-structure interaction problems. *Int'l J. Num. Methods Fluids* 67, 1175–1206.
- Wang, K., Gretarsson, J., Main, A., and Farhat, C., 2012. Computational algorithms for tracking dynamic fluid-structure interfaces in embedded boundary methods. *Int'l J. Num. Methods Fluids* 70, 515–535.

Yoon, J.W., Barlat, F., Dick, R.E., Chung, K., Kang, T.J., 2004. Plane stress function for Aluminum alloy sheets – Part II: FE formulation and its implementation. *Int'l J. Plasticity* 20, 495–522.

Figure Legends

- Fig. 1 Custom pressure testing facility used to conduct implosion experiments. (a) Photograph and (b) scaled schematic.
- Fig. 2 Cross section of pressure testing facility showing the illumination system, the high-speed video cameras, the pressure sensors and the data acquisition system.
- Fig. 3 (a) Schematic of shell specimen geometry for IMP69. (b) Positions of mid-span pressure sensors.
- Fig. 4 Schematic of the specimen and sensor support structure.
- Fig. 5 Sequence of high speed images that show the dynamic collapse of IMP69 (images taken at 0.16 ms intervals).
- Fig. 6 Signals recorded by four pressure sensors at mid-span of IMP69 illustrating the emanating pressure pulse. Numbered bullets correspond to images in Fig. 5.
- Fig. 7 Photographs showing the mode 4 collapse of IM69. (a) Side view and (b) end view.
- Fig. 8 Definitions of pressure signal characteristics.
- Fig. 9 (a) Positions of (a) mid-span and (b) off center pressure sensors.
- Fig. 10 Sequence of high speed images that show the dynamic collapse of IMP88.
- Fig. 11 Signals recorded by two pressure sensors at mid-span–(a) and (b)–and by two off-center sensors –(c) and (d)–for IMP88 illustrating the emanating pressure pulse. Numbered bullets correspond to images in Fig. 10.
- Fig. 12 Photographs showing the mode 2 collapse of IM88.
- Fig. 13 Control volume C_i (lighter lines) associated with vertex V_i of a tetrahedral mesh (heavier lines).
- Fig. 14 Surrogate fluid-structure interface in the vicinity of edge $V_i V_j$.
- Fig. 15 Truncated views of fluid CFD meshes for IMP69.

- Fig. 16 Calculated set of deformed configurations for IMP69 with color contours representing the radial displacement.
- Fig. 17 Comparison of measured and calculated pressure signals at two sensor locations for IMP69. Numbered bullets correspond to deformed configurations in Fig. 17.
- Fig. 18 Three-dimensional renderings of the calculated pressure fields around the collapsing shell for IMP69. Numbers correspond to times marked by bullets on the responses in Fig. 18.
- Fig. 19 Calculated collapsed configuration of shell of IMP69 at the time of first contact ($T \approx xxx$ ms)
- Fig. 20 Truncated views of fluid CFD meshes for IMP88.
- Fig. 21 Calulated set of deformed configurations for IMP88 with color contours representing the radial displacement.
- Fig. 22 Comparison of measured and calculated pressure signals at three sensor locations for IMP88. Numbered bullets correspond to deformed configurations in Fig. 22.
- Fig. 23 Three-dimensional renderings of the calculated pressure fields around the collapsing shell for IMP88. Numbers correspond to times marked by bullets on the responses in Fig. 23.
- Fig. 24 Calculated deformed configurations of shell of IMP88 at early stages of contact between the cell walls at mid-span.
- Fig. 25 Calculated final collapsed configuration of IMP88 with superimposed color contours of plastic equivalent strain.



(a)







(a)





F4









(a)



(b)













(a)



(b)









F15

























F21



















(a)



(b)



(c)

